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LETTERS' SECTION

The Arrow of Time

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Most speculations on the origin of the 'arrow of time', as well as most descriptions of experiments on 'time reversal', involve certain confusions that call for some conceptual analysis. We shall presently see that the catchall phrase 'the arrow of time' shelters three distinct ideas: time asymmetry, noninvariance under time reversal, and irreversibility. The first concerns a function, the second certain formulae, and the third certain processes.

The asymmetry of time consists in that durations are oriented intervals. More precisely, in any relational theory of time (Bunge, 1967, 1968), the time interval t between two events e and e', relative to a reference frame k, is $T(e,e',k) = t$. Now, time intervals are assumed to be infinitely divisible. That is, if e and e' are two events in k, there exists a third event e'', relative to the same frame k, such that $T(e,e',k) = T(e,e'',k) + T(e'',e',k)$. On setting $e'' = e'$, one obtains:

$$T(e,e',k) + T(e',e,k) = 0$$

and finally

$$T(e,e',k) = -T(e',e,k) \tag{1}$$

In short, the asymmetry or anisotropy of time (a) is a property of the (local) time function T, and (b) it consists in that, in any given frame, T is odd in the underlying events.

The preceding theorem of the relational theory of time clarifies the notion of time reversal. Indeed, changing the sign of the value t of T amounts to exchanging the order of the underlying events. For, if $t = T(e,e',k)$, then by (1) $-t = T(e',e,k)$. In other words, since $T(e,e',k) > 0$ if and only if e is earlier than e' (relative to k), the sign inversion of t corresponds to process reversal. Time reversal is not the inversion of the 'flow' of time (if only because there is no such flow) but a mathematical device for describing reversed processes, in particular processes with inverted velocities and spins. And the invariance (or noninvariance) under 'time reversal' is a property of certain statements (e.g. equations) containing the time variable (i.e. an arbitrary value of the time function T). T-invariance is neither a property of time nor a property of certain processes.

However, there is a relation between *T*-invariance and reversibility. The relation is this:

If a process is reversible, then the corresponding law statement is T-invariant (2)

Equivalently, if a law statement fails to be *T*-invariant, then it concerns an irreversible process. Thus, suppose a reaction is experimentally found to be reversible. Then this finding, jointly with the principle (2), entails that the corresponding law statement is *T*-invariant. If, on the other hand, the reaction proves irreversible, then nothing can be concluded about the corresponding law statement. Notwithstanding, some people conclude a 'violation' of time reversal invariance of the corresponding law statement. This is poor logic, and to speak in this connection of experiments in time reversal is bad metaphysics: time reversal is a purely conceptual operation and the time inverse of a reaction is nothing but the inverse of it. Both reactions, the given one and its inverse, proceed forward in time.

The obvious application to the recent speculations on time reversal linked to the decay of the neutral *K* meson, is the following. According to the CPT theorem, any process 'violating' the combined CP parity will 'violate' *T*-invariance as well—provided the process is adequately described by a CPT-invariant theory, that is. Assume then—though without too solid a ground—that kaons do satisfy a CPT-invariant theory, i.e. a Lorentz covariant local field theory. Then if their decay 'violates' the CP symmetry, it must also fail to be invariant under time reversal. But, according to our discussion, *T*-noninvariance is just an indication of irreversibility, not of an actual inversion of the 'arrow of time'. In short, all there is, is this: kaons cannot be reconstituted out of their decay products.

Notice that the converse of (2) is false. Indeed, while some irreversible processes are described by *T*-noninvariant laws, others are described with the help of *T*-invariant laws conjoined with certain subsidiary conditions that exclude or minimize the inverse processes. Thus, classical electromagnetic wave propagation is described by Maxwell's equations jointly with the Sommerfeld condition of outward radiation. Likewise, alpha decay can be accounted for by quantum mechanics (which is *T*-invariant) jointly with nuclear models involving, say, a potential barrier acting like a semipermeable membrane.

Since the converse of (2) is false, there is no equivalence between irreversibility and the 'violation' of time reversal invariance. Hence, it is as mistaken to define irreversibility in terms of *T*-noninvariance, as to read irreversibility in terms of a backward 'flow' of time. It is equally mistaken to attempt to 'define' time in terms of laws concerning irreversible processes. For one thing, some time concept must be at hand before an attempt is made to write down an irreversible law. For another, a law cannot act as a convention (e.g. a definition).

In conclusion (a) the asymmetry of time is a property of the local time function; (b) *T*-invariance concerns some law statements and it entails the possibility of process reversal, not the inversion of the direction of time; (c) *T*-invariance is necessary but insufficient for reversibility, and irreversibility can often be accounted for by *T*-invariant laws conjoined with certain subsidiary conditions (boundary conditions, constraints, etc.); (d) time can be characterised in terms of events and frames but it cannot be defined in terms of a special kind of law, among other reasons because a single concept of time is needed in various branches of physics both for overall coherence and for the interrelation of experimental results.

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Modalities and the Quantum Theory

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After many years of consideration the wave/particle duality problem in the quantum theory is still of interest. This letter seeks to sketch out this dual nature in terms of formal logic, and shows that one is drawn to the use of the modality operator to sort out the dilemma, which appears as a real mathematical problem in a system with two truth values.

Consider two experimental situations, C_1 and C_2, either of which imply the existence, E, of some entity, an electron for instance. Using '$\rightarrow$' as the usual implication connective in a two-valued logic, '$\vee$' and '$\wedge$' being 'and' and 'or' respectively, one has,

$$C_1 \rightarrow E \qquad C_2 \rightarrow E \qquad C_1 \vee C_2 \rightarrow E \qquad C_1 \wedge C_2 \rightarrow E \tag{1}$$

Suppose that in situation C_1 the entity has a significance best described by the existence P of a set of properties which one would associate closely with the idea of a classical particle, and that in situation C_2 the existence of a wave description, W, is best. Then

$$C_1 \rightarrow P \qquad C_2 \rightarrow W \tag{2}$$

Also, if the aforesaid set of particle properties occur, C_1 can occur.

$$P \rightarrow C_1 \qquad W \rightarrow C_2 \tag{3}$$

Thus, if one has a two-valued interpretation

$$W = C_1 \qquad P = C_2 \tag{4}$$

But

$$E \rightarrow P \vee W \qquad E \rightarrow P \wedge W \tag{5}$$